

SYMMETRY IN NATURE
COURSEWORK 2019–20

Due in at reception by 16:00 on Monday 16th March, 2020.

You may discuss the mathematics with your peers, and you may ask me questions, but what you hand in should of course be your own work. The work should not take more than 10 hours.

Marks may be deducted for poor presentation¹

Question 1 (On Euclidean transformations in the plane)

[6 marks]

- (a). Let $T \in E(n)$ be the transformation with Seitz symbol $(A \mid \mathbf{u})$; that is $f(\mathbf{x}) = A\mathbf{x} + \mathbf{u}$. Find the Seitz symbol for the inverse transformation T^{-1} .
 - (b). Interpret the transformation T with Seitz symbol $(r_{\pi/4} \mid (3, 1)^T)$ as a glide reflection. Illustrate the conclusion with a diagram showing the image of the origin under each of the steps of the glide reflection.
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Question 2 (Generators of \mathbb{Z}^2)

[10 marks]

Consider the group \mathbb{Z}^2

$$\mathbb{Z}^2 = \{(x, y) \mid x \in \mathbb{Z}, y \in \mathbb{Z}\}.$$

where the binary operation is addition.

- (a). Show that $\mathbf{a} = (1, 4)$, $\mathbf{b} = (2, 9)$ generate the group; that is

$$\mathbb{Z}^2 = \{n\mathbf{a} + m\mathbf{b} \mid m, n \in \mathbb{Z}\}.$$

[Hint: see problem 3.2 in the problem sheets.]

- (b). Denote by $\text{Mat}_n(\mathbb{Z})$ the set of all $n \times n$ matrices with integer entries. Let $A \in \text{Mat}_2(\mathbb{Z})$ be an invertible matrix. Show that $A^{-1} \in \text{Mat}_2(\mathbb{Z})$ if and only if $\det(A) = \pm 1$.
 - (c). Show that \mathbb{Z}^2 is generated by integer vectors (a, b) and (c, d) whenever $ad - bc = \pm 1$.
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Question 3 (The Octahedral group)

[4 marks]

Explain why the cube and the octahedron have the same symmetry group.

[A good diagram will help, but the explanation is important too, based on the duality between the two geometric objects. You will probably need to look this up on the internet.]

¹'presentation' means explaining the mathematics carefully and concisely, writing coherent and grammatically correct sentences and using careful notation. The work may be handwritten (legibly) or produced with a word processor (LaTeX recommended)